Math 70 9.3 Matrices to Solve Linear Systems Lesson 11
9.1 Solving Systems in 3 variables

9.3 Matrices to Solve Linear Systems (2x2 and 3x3)

Objectives

1) Write a linear system as a matrix
2) Interpret a matrix as a linear system
3) Find the reduced row-echelon form of a matrix using GC for 2X2 or 3X3 system.
   ⇒ This means "Solve system using RREF matrix command in GC."

4) Interpret RREF matrix result
   - consistent independent (no 0=0, no 0=1)
   - consistent dependent (yes 0=0, no 0=1)
   - inconsistent independent (no 0=0, yes 0=1)
   - inconsistent dependent (yes 0=0, yes 0=1)
5) Write solution of system of linear equations
   - ordered pair or triple
   - no solution (or empty set)
   - set notation for infinitely many solutions

9.1 Systems of Equations in 3 variables

* Do all of these problems using methods in 9.3! *

Math 70 9.1 Solving Systems in Three Variables & 9.3 Solving Linear Systems using Matrices

9.1 Systems in 3 variables: use matrix methods from 9.3!

9.3 Matrices to Solve Linear Systems

Objectives:

1. Write a linear system as a matrix
2. Interpret a matrix as a linear system
3. Find the Reduced Row-Echelon Form of a matrix using GC for a 2x2 or 3x3 system
   → This means "Solve system using RREF command on GC"
4. Interpret matrix result to classify 3x3 systems
   a. Graphical interpretation available, but not required
5. Recognize the RREF result for a consistent, independent 3x3 (no 0=0, no 0=1); ordered pair or ordered triple
6. Recognize the RREF result for a consistent, dependent 3x3 (has 0=0, no 0=1); set notation solution
7. Recognize the RREF result for inconsistent, independent 3x3 (has 0=1, but no 0=0); no solution
8. Recognize the RREF result for inconsistent, dependent 3x3 (has 0=1 AND 0=0); no solution
9. Write solutions using appropriate format: ordered pair/triple, no solution, or solution set.

CAUTION: If you type one number wrong when putting the matrix into your GC, you can get a very wrong result. To maximize credit, please write on your paper

- The matrix you typed in
- The matrix you got out
- Your solution

Write the system as a matrix.

1) $\begin{cases} -2y+3x=10 \\ 4x-3y=15 \end{cases}$

2) $\begin{cases} x=0 \\ y=-5 \end{cases}$

Solve and classify.

3) $\begin{cases} -2y+3x=10 \\ 4x-3y=15 \end{cases}$

4) $\begin{cases} 3x+\dfrac{y}{2}=2 \\ 6x+y=5 \end{cases}$

5) $\begin{cases} y=\dfrac{1}{7}x+3 \\ x-7y=-21 \end{cases}$

6) $\begin{cases} 2x+4y=3 \\ 4x-4z=-1 \\ y-4z=-3 \end{cases}$

7) $\begin{cases} 3x-3y+z=-1 \\ 3x-y-z=3 \\ -6x+2y+2z=-6 \end{cases}$

8) $\begin{cases} -6x+12y+3z=-6 \\ 2x-4y-z=2 \\ -x+2y+\dfrac{z}{2}=-1 \end{cases}$

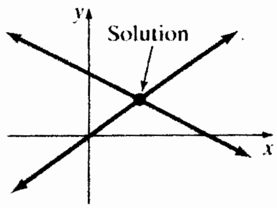
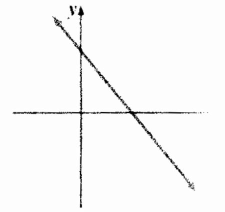
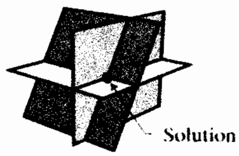
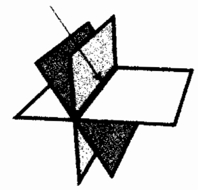
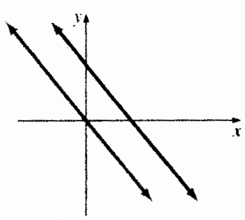
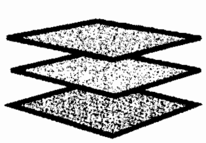
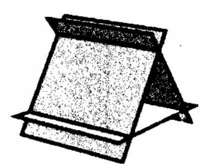
9) $\begin{cases} 6x-3y+12z=4 \\ -6x+4y-2z=7 \\ -2x+y-4z=3 \end{cases}$

10) $\begin{cases} -x+2y-3z=4 \\ 2x-4y+6z=8 \\ x-2y+3z=5 \end{cases}$

11) $\begin{cases} x-5y=0 \\ x-z=0 \\ -x+5z=0 \end{cases}$

12) $\begin{cases} x-4y-5z=35 \\ x-3y=0 \\ -y+z=-55 \end{cases}$

Math 70 Interpreting Results from Solving Linear Systems Using Matrices and RREF on GC

	INDEPENDENT $0=0$ does not appear in RREF	**DEPENDENT** $0=0$ appears in RREF
CONSISTENT System has solution(s)	2 variables $\begin{bmatrix}1&0&a\\0&1&b\end{bmatrix}$ Consistent Independent Solution (a,b) 	2 variables $\begin{bmatrix}1&a&b\\0&0&0\end{bmatrix}$ Consistent Dependent Solutions $\{(x,y): x+ay=b\}$ 
	3 variables $\begin{bmatrix}1&0&0&a\\0&1&0&b\\0&0&1&c\end{bmatrix}$ Consistent Independent Solution (a,b,c) 	3 variables Solutions (line) $\begin{bmatrix}1&0&a&b\\0&1&c&d\\0&0&0&0\end{bmatrix}$ Consistent Dependent Solutions $\{(x,y,z) \mid x=b-az, y=d-cz, z\in R\}$ or $(b-az, d-cz, z)$ where $z\in R$ 
		3 variables $\begin{bmatrix}1&a&b&c\\0&0&0&0\\0&0&0&0\end{bmatrix}$ Solutions (plane) Consistent Dependent Solutions $\{(x,y,z) \mid x+ay+bz=c\}$ 
INCONSISTENT Systems has no solution. 0=1 appears in RREF	2 variables $\begin{bmatrix}1&0&a\\0&0&1\end{bmatrix}$ Inconsistent (Independent) No solution { } 	2 variables Inconsistent Dependent is not possible
	3 variables $\begin{bmatrix}1&0&a&b\\0&1&c&d\\0&0&0&1\end{bmatrix}$ Inconsistent Independent No Solution { } 	3 variables $\begin{bmatrix}1&a&b&c\\0&0&0&1\\0&0&0&0\end{bmatrix}$ Inconsistent Dependent No Solution { } 

① Write the system $\begin{cases} -2y+3x=10 & (A) \\ 4x-3y=15 & (B) \end{cases}$ as an expanded matrix.

(same as #5)

Step 1: Write the system in standard form. Notice (A) is out of order!

$$\begin{cases} 3x-2y=10 & (A) \\ 4x-3y=15 & (B) \end{cases}$$

step 2: Use only the coefficients, no variables. Write coefficients in order, one row per equation.

$$\underline{\underline{3}}\,x + \underline{\underline{(-2)}}\,y = \underline{\underline{10}} \Rightarrow \quad 3 \quad -2 \quad 10$$

$$\underline{\underline{4}}\,x + \underline{\underline{(-3)}}\,y = \underline{\underline{15}} \Rightarrow \quad 4 \quad -3 \quad 15$$

Step 3: Add brackets

$$\boxed{\begin{bmatrix} 3 & -2 & 10 \\ 4 & -3 & 15 \end{bmatrix}}$$

← this is how we will type the question into the GC.

② Write the system $\begin{cases} x=0 \\ y=-5 \end{cases}$ as an expanded matrix.

(same as #5)

Step 1: Write in standard form, adding zeros for any missing terms

$$\begin{cases} \underline{\underline{1}}x + \underline{\underline{0}}y = \underline{\underline{0}} \\ \underline{\underline{0}}x + \underline{\underline{1}}y = \underline{\underline{-5}} \end{cases} \Rightarrow \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & -5 \end{matrix}$$

Step 2: Write coefficients only, one row per equation, in brackets

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -5 \end{bmatrix}}$$

← this is how the GC will display the solution to the system. It's called Reduced Row Echelon Form (RREF)

Math 70 5/e 4.4

(3) Solve the system $\begin{cases} -2y+3x=10 \\ 4x-3y=15 \end{cases}$ using a matrix and the RREF command on your GC.

(same as #5, 6, 7)

<u>step 1</u>: Write system in standard form, then write the expanded matrix.

<u>CAUTION</u>: Always show this step on paper! * increases odds you'll do it correctly * gives me something to reproduce on my GC * will earn partial credit for incorrect answers

$$\begin{cases} 3x-2y=10 \\ 4x-3y=15 \end{cases} \Rightarrow \begin{bmatrix} 3 & -2 & 10 \\ 4 & -3 & 15 \end{bmatrix}$$

<u>step 2</u>: Use GC to solve

$\boxed{2nd}\ \boxed{x^{-1}}$ = MATRIX

NAMES	MATH	EDIT

⇐ 3 menus.

<u>1st</u>: We'll use $\boxed{\text{EDIT}}$ to type in the matrix

<u>2nd</u>: We'll use $\boxed{\text{MATH}}$ to get the RREF calculation.

<u>3rd</u>: We'll use $\boxed{\text{NAMES}}$ to tell the GC which matrix to use.

⇒

M70 4.4

3 con't

Press ▷ Twice to get the $\boxed{\text{EDIT}}$ menu.
Press $\boxed{\text{ENTER}}$ to select matrix [A].

$\boxed{\text{MATRIX[A]} \quad \_\_ \times \_\_}$

3 enter
2 enter

- - - - - - - - - -

3 enter
-2 enter
10 enter

- - - - - - - - - -

4 enter
-3 enter
15 enter

$\boxed{\text{2nd}}$ $\boxed{\text{MODE}}$ = QUIT

first enter the number of rows = 2
(rows = equations)

then enter the number of columns = 3
(columns = variables + 1)

then enter the coefficients of the first equation

CAUTION: Must exit the $\boxed{\text{EDIT}}$ mode or your matrix will be messed up later.

$\boxed{\text{2nd}}$ $\boxed{x^{-1}}$ = MATRIX

Press ▷ once to get the $\boxed{\text{MATH}}$ menu.
Scroll down to next screen
B. rref
$\boxed{\text{ENTER}}$

$\boxed{\text{rref(}}$

Calculator is waiting for a matrix name.

$\boxed{\text{2nd}}$ $\boxed{x^{-1}}$ = MATRIX

Press 1 or $\boxed{\text{ENTER}}$ to select matrix [A].
Press $\boxed{\text{ENTER}}$ to complete the calculation.

(3) con't

$$\boxed{\begin{array}{l}\text{rref (A)}\\ [[1 \quad 0 \quad 0]\\ \quad [0 \quad 1 \quad -5]]\end{array}}$$

This means matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -5 \end{bmatrix}$

Translate this to variables and a system of eq'ns:

$$\begin{cases} 1x + 0y = 0 \\ 0x + 1y = -5 \end{cases}$$

Simplify $x = 0$
$y = -5$

solve ↓ classify ↓

Write as an ordered pair $\boxed{(0, -5)}$ $\boxed{\text{consistent independent}}$

CAUTION: If you type one number in wrong, you can get a very wrong result.

To maximize credit, please write
- The matrix you put in
- The matrix you got out
- your solution

on your paper.

Math 70 5/e 4.2

(4) Explore: Solve by GC RREF, algebra, graphing on GC. Classify and summarize.

$$\begin{cases} 3x + \dfrac{y}{2} = 2 & \text{(A)} \\ 6x + y = 5 & \text{(B)} \end{cases}$$

Note: $\frac{y}{2}$ means $\frac{1}{2}y$ or $0.5y$

matrix is 2×3

$$\begin{bmatrix} 3 & \frac{1}{2} & 2 \\ 6 & 1 & 5 \end{bmatrix} \xrightarrow[\text{RREF}]{} \begin{bmatrix} 1 & 1/6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Translate this RREF matrix result back to a 2×2 system of equations:

$$\begin{cases} 1x + \frac{1}{6}y = 0 \\ 0x + 0y = 1 \end{cases}$$

Simplify:

$$\begin{cases} x + \frac{1}{6}y = 0 \\ \qquad 0 = 1 \end{cases}$$

This result is crucial! $0 \neq 1$, yet solving the equation by the elimination method gives this result. What does this mean?

Use algebra

Solve: (Elimination shown, but substitution ok, too.)

$3x + \frac{1}{2}y = 2$ (A) $\times (-2)$

$6x + y = 5$ (B)

$$\begin{array}{r} -6x - y = -4 \\ 6x + y = 5 \\ \hline 0 + 0 \neq 1 \end{array}$$

"solve" "classify"

It means $\boxed{\text{no solution}}$ $\boxed{\text{inconsistent}}$

Math 70 5/e 4.2

To graph on GC, isolate y:

$3x + \frac{y}{2} = 5$	$6x + y = 5$
$6x + y = 10$	$y = -6x + 5$
$y = -6x + 10$	

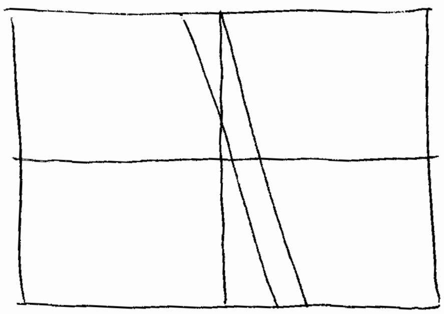
slope $= -6$ ← same slope → slope $= -6$

y-int $= 10$ ← different y-ints → yint $= 5$

$\boxed{Y=}$ $y_1 = -6x + 10$

$y_2 = -6x + 5$

$\boxed{\text{ZOOM}}$ $\boxed{6}$



I. parallel lines do not intersect — graph, same slope diff yint

II $0 \neq 1$ — RREF, algebraic solve

III. system is inconsistent — classify

IV. system has $\boxed{\text{no solution}}$ or $\boxed{\phi}$ OR $\boxed{\{\ \}}$ — solve

$$\boxed{\underline{\text{CAUTION}}:\ \emptyset \text{ or } \{\ \} \text{ but } \underline{\text{not}}\ \{\phi\}}$$

Math 70 5/e 4.2

5 Explore

$$\begin{cases} y = \frac{1}{7}x + 3 & \text{A} \\ x - 7y = -21 & \text{B} \end{cases}$$

Standard form:

$$y = \frac{1}{7}x + 3 \qquad \text{A}$$

$$\left.\begin{aligned} -\tfrac{1}{7}x + y &= 3 \\ x - 7y &= -21 \end{aligned}\right\} \quad \begin{aligned} &\text{A} \\ &\text{B} \end{aligned}$$

matrix

$$\begin{bmatrix} -1/7 & 1 & 3 \\ 1 & -7 & -21 \end{bmatrix} \xrightarrow[\text{RREF}]{} \begin{bmatrix} 1 & -7 & -21 \\ 0 & 0 & 0 \end{bmatrix}$$

Translate this RREF matrix result back to a 2x2 system of equations.

$$\begin{cases} 1x - 7y = -21 \\ 0x + 0y = 0 \end{cases}$$

Simplify.

$$\begin{cases} x - 7y = -21 \\ 0 = 0 \end{cases}$$

← This result is crucial! 0=0, true, but what does this mean?

Use algebra

Solve: (Substitution shown, but elimination OK, too.)

A ⇒ B $\quad x - 7\left(\frac{1}{7}x + 3\right) = -21 \qquad$ replace y

$x - x - 21 = -21 \qquad$ distribute

$-21 = -21$

$0 = 0$

It means infinitely many ordered pairs are on both lines, are solutions.

Math 70 5/e 4.2

To graph on GC, isolate y

(A) $y = \frac{1}{7}x + 3$

(B) $x - 7y = -21$

$$\frac{-7y}{-7} = \frac{-x}{-7} - \frac{21}{-7}$$

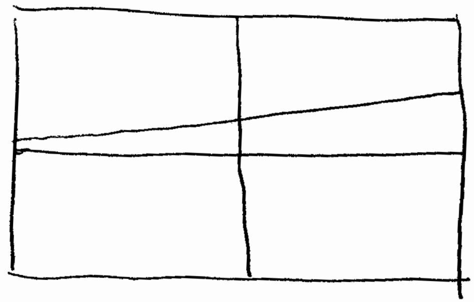
$$y = \frac{1}{7}x + 3$$ same as (A)

$\boxed{y=}$

$y_1 = \boxed{1/7}\, x + \boxed{3}$

$y_2 = \boxed{1/7}\, x + \boxed{3}$

same slope   same y-int



same line means any ordered pair on (A) is also on (B)

⇒ any ordered pair on the line is a solution.

I. two equations have same graph   graph, same slope & same y-int

II. 0=0   RREF, algebraic solve

III. system is consistent dependent   classify

IV. system has infinitely many solutions

write as a solution set:

$$\boxed{\{(x,y) \mid x - 7y = -21\}}$$

solve

classify 2

$\boxed{\text{consistent dependent}}$

end of set

"The set of all   ordered pairs   such that   this equation is true."

CAUTION: Do not say "all real numbers" → this is a one-dimensional (x only) answer. Plus, not all pairs work. (0,0), for example, is not a solution

Math 70 5/e 4.2

(6) Solve using matrices on GC.

$$\begin{cases} 2x + 4y = 3 & \text{(A)} \\ 4x - 4z = -1 & \text{(B)} \\ y - 4z = -3 & \text{(C)} \end{cases}$$

Write in standard form, using placeholders for missing variables:

$$\begin{cases} 2x + 4y + 0z = 3 \\ 4x + 0y - 4z = -1 \\ 0x + 1y - 4z = -3 \end{cases}$$

This system has 3 rows and 4 columns.
Edit the dimensions of the augmented matrix to be 3x4. (Always $\underline{\underline{\text{rows}}}$ x columns)

$$\begin{bmatrix} 2 & 4 & 0 & 3 \\ 4 & 0 & -4 & -1 \\ 0 & 1 & -4 & -3 \end{bmatrix}$$

RREF as before

$$\begin{bmatrix} 1 & 0 & 0 & .611111\ldots \\ 0 & 1 & 0 & .444444\ldots \\ 0 & 0 & 1 & .861111\ldots \end{bmatrix}$$

Convert to fractions

$\boxed{\text{MATH}}$ $\boxed{\text{ENTER}}$

$$\boxed{\text{Ans}>\text{Frac}}$$

"Ans" means the answer in the previous calculation.

$\boxed{\text{ENTER}}$

$$\begin{bmatrix} 1 & 0 & 0 & 11/18 \\ 0 & 1 & 0 & 4/9 \\ 0 & 0 & 1 & 31/36 \end{bmatrix}$$

much better! Exact answers, no rounding.

Math 70 5/e 4.2

Translate back to a linear system of equations

$$\begin{cases} 1x + 0y + 0z = 11/18 \\ 0x + 1y + 1z = 4/9 \\ 0x + 0y + 1z = 31/36 \end{cases}$$

Simplify

$$\begin{cases} x = 11/18 \\ y = 4/9 \\ z = 31/36 \end{cases}$$

Solve

Write as an ordered triple $\boxed{\left(\frac{11}{18}, \frac{4}{9}, \frac{31}{36}\right)}$

classify?

$\boxed{\text{consistent independent}}$

Math 70 5/e 4.2

Solve each linear system using matrices on your GC.

(7) $\begin{cases} 3x - 3y + z = -1 \\ 3x - y - z = 3 \\ -6x + 2y + 2z = -6 \end{cases}$

augmented matrix: $\begin{bmatrix} 3 & -3 & 1 & -1 \\ 3 & -1 & -1 & 3 \\ -6 & 2 & 2 & -6 \end{bmatrix}$

RREF > frac

$\begin{bmatrix} 1 & 0 & -2/3 & 5/3 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow 0=0$

no 0=1 in this RREF

Translate back to linear system:

$\begin{cases} x - \frac{2}{3}z = \frac{5}{3} \\ y - z = 2 \\ 0 = 0 \end{cases}$

$\Rightarrow$ solve for x $\quad x = \frac{2}{3}z + \frac{5}{3}$

$\Rightarrow$ solve for y $\quad y = z + 2$

Infinitely many solutions:

solve

$$\boxed{\{(x,y,z) \mid x = \tfrac{2}{3}z + \tfrac{5}{3},\ y = z+2,\ z \text{ is a real \#}\}}$$

The phrase "z is a real number"
can be abbreviated "$z \in \mathbb{R}$"
which means "z is an element of the set of real numbers".

classify

This is a $\boxed{\text{consistent dependent}}$ system
where the solution is a line.

Math 70 5/e 4.2

(8) $\begin{cases} -6x + 12y + 3z = -6 \\ 2x - 4y - z = 2 \\ -x + 2y + \frac{z}{2} = -1 \end{cases}$

$$= \left[\begin{array}{cccc} -6 & 12 & 3 & -6 \\ 2 & -4 & -1 & 2 \\ -1 & 2 & \frac{1}{2} & -1 \end{array}\right]$$

$\Rightarrow$ RREF

$$= \left[\begin{array}{cccc} 1 & -2 & -.5 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\begin{cases} x - 2y - \frac{1}{2}z = 1 \\ 0 = 0 \\ 0 = 0 \end{cases}$ } These mean $\boxed{\text{consistent dependent}}$, infinitely many solutions.

classify

$$\boxed{\{(x,y,z) \mid x - 2y - \tfrac{1}{2}z = 1\}}$$ solve

(9) $\begin{cases} 6x - 3y + 12z = 4 \\ -6x + 4y - 2z = 7 \\ -2x + y - 4z = 3 \end{cases}$

matrix 3×4

$$\left[\begin{array}{cccc} 6 & -3 & 12 & 4 \\ -6 & 4 & -2 & 7 \\ -2 & 1 & -4 & 3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc} 1 & 0 & 7 & 0 \\ 0 & 1 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \leftarrow \text{Last row means } 0 \neq 1 \text{ False}$$

solve

$\boxed{\text{No Solution}}$

classify

This is an $\boxed{\text{inconsistent independent}}$ system

Math 70 5/e 4.2

(10)
$$\begin{cases} -x + 2y - 3z = 4 \\ 2x - 4y + 6z = 8 \\ x - 2y + 3z = 5 \end{cases}$$

augmented matrix

$$\begin{bmatrix} -1 & 2 & -3 & 4 \\ 2 & -4 & 6 & 8 \\ 1 & -2 & 3 & 5 \end{bmatrix}$$

RREF

$$\begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

translate back to system of equations:

$$\begin{cases} x - 2y + 3z = 0 \\ 0 = 1 \quad \Leftarrow \text{false} \\ 0 = 0 \end{cases}$$

This is an $\boxed{\text{inconsistent}\ \ \text{dependent}}$ system.

$\boxed{\text{no solution}}$ ← solve

← classify

Math 70 5/e 4.2

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$$\begin{cases} x - 5y = 0 \\ x - z = 0 \\ -x + 5z = 0 \end{cases}$$

$$\begin{bmatrix} 1 & -5 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ -1 & 0 & 5 & 0 \end{bmatrix}$$

$\Rightarrow$ RREF

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

translates to

$x = 0$
$y = 0$
$z = 0$

$\boxed{(0, 0, 0)}$ ← solve

$\boxed{\text{consistent independent}}$

12

$$\begin{cases} x - 4y - 5z = 35 \\ x - 3y = 0 \\ -y + z = -55 \end{cases}$$

$$\begin{bmatrix} 1 & -4 & -5 & 35 \\ 1 & -3 & 0 & 0 \\ 0 & -1 & 1 & -55 \end{bmatrix}$$

$\Rightarrow$ RREF

$$\begin{bmatrix} 1 & 0 & 0 & 120 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & -15 \end{bmatrix}$$

$\boxed{(120, 40, -15)}$ ← solve

$\boxed{\text{consistent independent}}$

↑ classify

Name ______________________________

Date ______________________________

TI-84+ GC 24 Solving Systems of Linear Equations with Matrices

Objectives: Review standard form of a linear equation and definition of system of linear equations
Identify coefficients of a linear system
Write augmented matrix and its dimension for a system of linear equations
Recognize the solution of a system when written as augmented matrix
Use GC to find the Reduced Row Echelon Form (RREF) of an augmented matrix

> Recall: The standard form of a linear equation is $ax+by=c$, where a,b,c are coefficients.
> A system of linear equations is a group of linear equations that must be solved together, so that the solution of the system is a solution of every linear equation in the system.

Example 1: Write this linear equation in standard form: $y=-\frac{5}{6}x+\frac{7}{12}$

Clear fractions by multiplying by LCD 12: $12y=-10x+7$

Add 10x to both sides: $10x+12y=7$

Answer: $\underline{10x+12y=7}$

Example 2: Identify the coefficients of the linear equation: $-3x+\frac{2}{5}y=9.2$

Answer: $\boxed{-3,\ \frac{2}{5},\ 9.2}$

> A powerful way to use the GC to solve a linear system is to use an abbreviated way of writing a linear system, called the augmented coefficient matrix. If we write every linear equation in standard form, then we know where the variables and = belong, even if we don't write them.

Example 3: Write the augmented coefficient matrix for $\begin{cases}2x-3y=6\\-7x+5y=-1\end{cases}$.

Step 1: Check that the equations are in standard form. (They are.)

Step 2: Abbreviate the system by writing only the coefficients, in the same order and placement:

Answer: $\begin{bmatrix}2 & -3 & 6\\-7 & 5 & -1\end{bmatrix}$.

> IMPORTANT: There must be a number in every place in the matrix. Use 0 for missing terms or coefficients.

Example 4: $\begin{cases}5x=9\\2x-6y=-1\end{cases}$

Notice that 5x = 5x + 0y

Answer: $\begin{bmatrix}5 & 0 & 9\\2 & -6 & -1\end{bmatrix}$

A matrix with 1s in the diagonal places (starting in the upper left corner) and 0s above and below the diagonal is in Reduced Row Echelon Form (RREF).
CAUTION: Do not confuse RREF with REF. REF will NOT solve the system completely.

Example 5: $(9.2,-3.5)$ is the solution to a linear system. Write the solution as two equations (a linear system) and write the system as an augmented matrix.

Answer: The system of equations is $\begin{cases} x = 9.2 \\ y = -3.5 \end{cases}$. The matrix is $\begin{bmatrix} 1 & 0 & 9.2 \\ 0 & 1 & -3.5 \end{bmatrix}$, and is in RREF.

Since the GC will use only the augmented coefficient matrices, we need to work backwards also.

Example 6: Write the linear system for $\begin{bmatrix} -2 & 5 & 6 \\ 3 & -1 & 7 \end{bmatrix}$. Use x and y (and z if needed.)

Answer: $\begin{cases} -2x + 5y = 6 \\ 3x - y = 7 \end{cases}$

The GC will need to know the size of the matrix. The size of a matrix is measured as: (Number of rows) by (Number of columns) and is called the dimension of the matrix.
IMPORTANT: The number of rows is always listed first. (R x C)

Example 7: Find the dimension of $\begin{bmatrix} -2 & 5 & 6 \\ 3 & -1 & 7 \end{bmatrix}$.

$\begin{bmatrix} -2 & 5 & 6 \\ 3 & -1 & 7 \end{bmatrix}$ has 2 rows (horizontal) $\begin{bmatrix} --- \\ --- \end{bmatrix}$ and three columns (vertical) $\begin{bmatrix} | & | & | \\ | & | & | \end{bmatrix}$.

It is a 2x3 matrix, which we pronounce "two by three". Answer: 2x3

The GC will use the addition (also called elimination) method that you know. It multiples a row of the matrix by a useful number and adds it to another row, just as you multiply equations by useful numbers and add to eliminate variables. When the GC has eliminated a variable, a zero appears in the matrix. The GC will show you only the final answer.

Step 1: On paper, write the linear system as an augmented matrix and determine the dimension.
Step 2: Go to MATRIX – EDIT, select a matrix name, then input the dimension and coefficients.
IMPORTANT: Exit MATRIX – EDIT and return to the basic calculating screen.
Step 3: Go to MATRIX – MATH, select RREF, which is option B, after options 1-9 and A, on the second screen. Press ENTER to select RREF.
Step 4a: Go to MATRIX – NAMES and select the matrix you edited in step 2. Press ENTER to select this matrix, and return to the basic calculating screen.
Optional Step 4b: If you suspect fractional answers, select MATH - >FRAC - ENTER. This can also be done after Step 5 if needed.
Step 4c: Press ENTER again to complete the RREF calculation.
Step 5: Translate the row-reduced matrix back into equations, and write the ordered pair or triple.

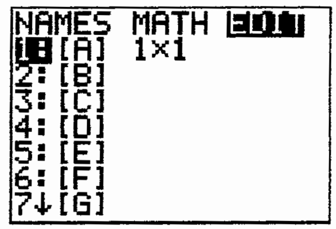
Example 8: Solve $\begin{cases} 2x - 3y = -30 \\ -4x + y = 20 \end{cases}$ using your GC.

Step 1: On paper, write the linear system as an augmented matrix and determine the dimension.
$\begin{cases} 2x - 3y = -30 \\ -4x + y = 20 \end{cases}$ is in standard form. The augmented matrix is $\begin{bmatrix} 2 & -3 & -30 \\ -4 & 1 & 20 \end{bmatrix}$, which is 2x3.

Step 2: Go to MATRIX – EDIT, select a matrix name, then input the dimension and coefficients.

Above the x^{-1} is a 2nd function which opens the MATRIX menu. MATRIX has three menus within it: NAMES, MATH, and EDIT. We'll use all three of these menus to solve the system.

Go to MATRIX, and select EDIT: 2nd x^{-1} ▶ ▶

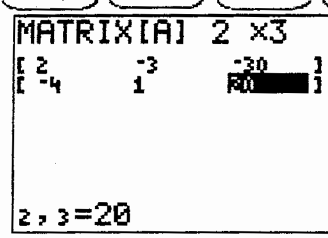


The first matrix in the list, called [A] is already highlighted, so let's choose it. ENTER

The GC is waiting for the dimensions of the matrix: 2 ENTER 3 ENTER.

The GC is waiting for the coefficients of the matrix. The GC moves automatically across the rows.
2 ENTER (-) 3 ENTER (-) 3 0 ENTER (-) 4 ENTER

1 ENTER 2 0 ENTER



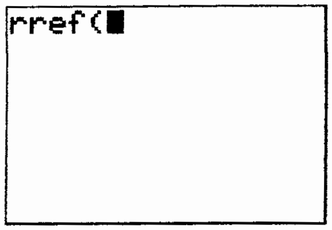
IMPORTANT: Exit MATRIX – EDIT and return to the basic calculating screen. 2nd MODE

Step 3: Go to MATRIX – MATH, select RREF, which is option B, after options 1-9 and A, on the second screen. Press ENTER to select RREF.

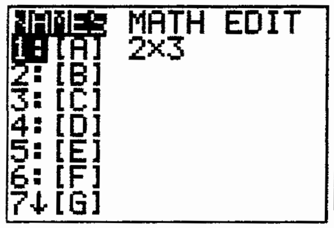
2nd x^{-1} ▶

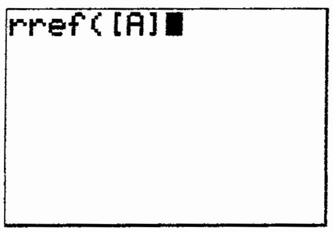
▼ 11 times (or ▲ 5 times)

Example 8 (continued):

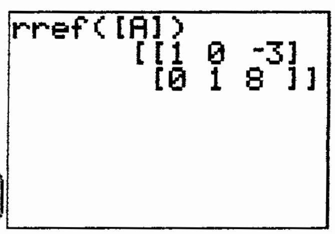


Step 4a: Go to MATRIX – NAMES and select the matrix you edited in step 2. Press ENTER to select this matrix, and return to the basic calculating screen. (We'll skip Optional Step 4b.)

Matrix [A] is already highlighted.




Step 4c: Press ENTER again to complete the RREF calculation.
The GC is waiting for you to close the parentheses (which isn't always necessary) and ENTER.



Step 5: Translate the row-reduced matrix back into equations, and write the ordered pair or triple.

$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 8 \end{bmatrix}$ means $\begin{cases} x=-3 \\ y=8 \end{cases}$

Answer: (-3,8)

CAUTION: Reduced row echelon form of an augmented matrix only applies to *linear* systems. This procedure cannot be used to solve *nonlinear* systems.